Eur Phys J C **33**, s01, s244–s246 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2003-03-122-5

EPJ C direct
electronic only

QCD factorisation in radiative B decays

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Received: 14 October 2003 / Accepted: 24 October 2003 /
Published Online: 5 November 2003 –

Abstract. We study how, in the heavy-quark limit and at one loop, the amplitude of the radiative decays $B \to \gamma\ell\nu_\ell, B \to \gamma\gamma, B \to \gamma\ell^+\ell^-$ factorise, i.e., they can be written as a convolution of a (perturbative) hard-scattering amplitude and the (nonperturbative) light-cone distribution amplitude of the B-meson. Using the framework of the Soft-Collinear Effective Theory, large logarithms can be resummed and the amplitudes of the 3 decays are shown to differ from each other only through the dynamics above M_B.

Recently, the powerful framework of QCD factorisation [1] was introduced for nonleptonic exclusive B-meson decays: in the heavy-quark limit, their amplitudes can be factorised, i.e. expressed in terms of process-dependent kernels, which are computable perturbatively, and universal nonperturbative objects, such as form factors and light-cone distribution amplitudes (LCDA). In order to understand better the structure of higher-order corrections, radiative B-decays are very useful, since they involve only one nonperturbative object, namely the B-meson LCDA [2]. We inverstigate here QCD factorisation for the decays $B \to X\gamma$ where $X = \ell\nu, \ell^+\ell^-, \gamma$ [3,4], in the case where the energy of both the photon and X is large and of order M_B (we neglect the mass of the lepton).

1 $B \to \gamma\ell\nu$

1.1 Factorisation at tree level

The hadronic matrix element for the decay $B \to \gamma\ell\nu_\ell$ can be written in terms of two form factors F_V and F_A:

$$\frac{1}{\sqrt{4\pi\alpha}}\langle\gamma(\varepsilon^*,q)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = \qquad (1)$$
$$\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}v^\rho q^\sigma F_V(E_\gamma) + i[\varepsilon^*_\mu(v\cdot q) - q_\mu(v\cdot\varepsilon^*)]F_A(E_\gamma)\,.$$

We work in the B-meson rest frame and we introduce light-cone coordinates $l = (l_+, l_-, \mathbf{l}_\perp)$, defined by $l_\pm = (l_0 \pm l_3)/\sqrt{2}$ and $\mathbf{l}_\perp = (l_1, l_2)$, so that

$$p = (M_B/\sqrt{2}, M_B/\sqrt{2}, \mathbf{0}_\perp) \qquad q_2 = (0, q_-, \mathbf{0}_\perp)\,. \qquad (2)$$

We define the light-cone distribution amplitude (LCDA) of a state H which contains the b-quark by

$$\Phi^H_{\alpha\beta}(\tilde{k}_+) = \int dz_-\ e^{i\tilde{k}_+ z_-}\ \langle 0|\bar{u}_\beta(z)[z,0]b_\alpha(0)|H\rangle|_{z_+,z_\perp=0}\,, \qquad (3)$$

[a] LPT-ORSAY-03-74

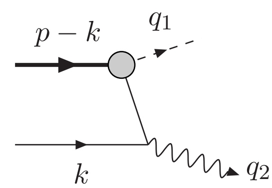

Fig. 1. Lowest-order diagram at leading twist contributing to the process $b\bar{u} \to \gamma X$, where $X = \ell\nu_\ell$, $\ell^+\ell^-$ or γ and is represented by the dashed line. The *thick line* represents the b-quark and the *thin line* a light quark. The *grey circle* represents the operator responsible for the $b \to u$ transition

where u, b are the quark fields and α, β are spinor labels. $[z, 0]$ denotes the gluon path-ordered exponential. F^H_μ is the matrix element of the weak $b \to u$ current,

$$F^H_\mu \equiv \langle\gamma(\varepsilon^*,q)|\bar{u}\gamma_{\mu\,L}b|H\rangle\,, \quad \gamma_{\mu\,L} \equiv \gamma_\mu(1-\gamma_5)\,. \qquad (4)$$

The question is whether, up to one-loop order in perturbation theory, the matrix element can be written in the *factorised* form [2,3]

$$F^H_\mu = \int \frac{d\tilde{k}_+}{2\pi}\Phi^H_{\alpha\beta}(\tilde{k}_+)T_{\beta\alpha}(\tilde{k}_+)\,, \qquad (5)$$

where the hard-scattering amplitude T does not depend on the external state (H) and is a function of hard scales only. We stress the distinction between $\tilde{k}_+$ and the kinematical variables of the initial state H. In general, the LCDA $\Phi^H_{\alpha\beta}$ depends on the latter in a complicated and non-perturbative way. The question we investigate here is whether this dependence matches that in F^H_μ so that (5) holds with a single convolution variable.

Since T is independent of the external state H, we can choose any convenient external state to compute it: here, we take a pair of free on-shell quarks $H = |b^S(p-k)u(k)\rangle$. At tree level, the diagram in Fig. 1 and the crossed diagram arise, but the latter is higher-twist due to the $1/M_B$-suppression of the off-shell internal quark propagator. In order to evaluate the hard-scattering amplitude we proceed in the standard way: compute the matrix element,

$$F_\mu^{(0)\,b\bar u} = -\frac{e_u}{2q_- k_+}\ \left\{\bar v^s(k)\,\not\epsilon^*\,\not q_2\,\gamma_{\mu\,L}\,u^S(p-k)\right\}, \tag{6}$$

then evaluate the LCDA at tree level,

$$\Phi^{b\bar u}_{\alpha\beta}(\tilde k_+) = 2\pi\delta(k_+ - \tilde k_+)\ \bar v^s_\beta(k)\,u^S_\alpha(p-k), \tag{7}$$

and finally combine the two to deduce the hard-scattering amplitude:

$$F_\mu^{(0)\,b\bar u} = \int \frac{d\tilde k_+}{2\pi}\,\Phi^{(0)\,b\bar u}_{\alpha\beta}(\tilde k_+)\,T^{(0)}_{\beta\alpha}(\tilde k_+), \tag{8}$$

with the hard-scattering amplitude at tree level,

$$T^{(0)}_{\beta\alpha}(\tilde k_+) = -\frac{e_u}{2q_-\tilde k_+}[\not\epsilon^*\not q_2\gamma_{\mu\,L}]_{\beta\alpha}. \tag{9}$$

As expected the hard-scattering amplitude depends only on $\tilde k_+$ (through the *hard* variable $q_2\cdot\tilde k$). One can check that the result (9) does not depend on the choice of the external state by taking $H = |q\bar q g\rangle$ [3].

Once T is known, one can switch back to the B-meson as the external state. The Dirac structure of $\Phi^B_{\alpha\beta}$ allows one to define two (scalar) LCDA Φ^B_+ and Φ^B_- [5], which leads to the following result for the form factors [3]:

$$F_{A,V} = -\frac{f_B M_B\,Q_u}{3\sqrt{2}\,E_\gamma}\int_0^\infty d\tilde k_+\,\frac{\Phi^B_+(\tilde k_+)}{\tilde k_+} + O\left(\alpha_s, \frac{1}{M_B}\right) \tag{10}$$

1.2 Factorisation at one loop

If factorisation holds at higher orders of perturbation theory, one should obtain:

$$\begin{aligned} F_\mu^H &= F_\mu^{(0)\,H} + F_\mu^{(1)\,H} + \cdots = \Phi^H \otimes T \qquad (11)\\ &= [\Phi^{(0)\,H}\otimes T^{(0)}] + [\Phi^{(0)\,H}\otimes T^{(1)} + \Phi^{(1)\,H}\otimes T^{(0)}] + \ldots \end{aligned}$$

where $\otimes$ denotes the convolution, and the superscripts indicate the power of α_s. The hard-scattering kernels $T^{(n)}$ contain only hard scales, whereas the distribution amplitudes $\Phi^{(n)}$ absorb all the soft effects.

This decay is a three-scale process, with a large scale m_b, a small scale Λ_{QCD} and an intermediate scale $(2q\cdot\tilde k)^{1/2}$, which will correspond to the factorisation (separation) scale. We work in the Feynman gauge and the $\overline{\mathrm{MS}}$ scheme: we denote μ_R the (renormalisation) scale arising in the computation F_μ^H and μ_F the (factorisation) scale for $\Phi^H\otimes T$. A natural choice is $\mu_R = O(m_b)$ and $\mu_F^2 = O(m_b\Lambda_{\mathrm{QCD}})$. Moreover, since the LCDA of the B-meson involves only dynamics well below M_B, we choose to define it in HQET rather than in QCD.

As in sec. 1.1, we evaluate $T^{(1)}$ by taking $H = |b^S(p-k)\,\bar u^s(k)\rangle$ with $k = O(\Lambda_{\mathrm{QCD}})$. Equation (11) yields:

$$\Phi^{(0)\,H}\otimes T^{(1)} = F_\mu^{(1)\,H} - \Phi^{(1)\,H}\otimes T^{(0)}, \tag{12}$$

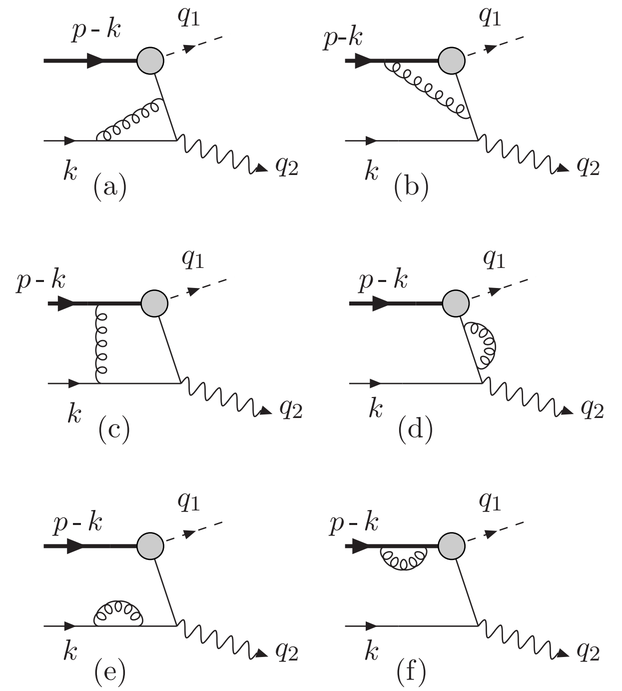

Fig. 2. One-loop leading-twist diagrams for $b\bar u \to X\gamma$

so that, at one-loop order we need to evaluate both $\Phi^{(1)\,H}$ and $F_\mu^{(1)\,H}$. The corresponding diagrams are shown in Fig. 2. There are IR singularities in both terms on the right-hand side of (12), but they should cancel in the difference for factorisation to hold. For this process, this cancellation occurs diagram by diagram, and we outline now a few specificities of each diagram.

The electromagnetic vertex diagram, Fig. 2a, leads to a mass singularity for $F_\mu^{(1)}$ from the region collinear to k. However, $\Phi^{(1)}\otimes T^{(0)}$ exhibits exactly the same singularity – its expression corresponds to $F_\mu^{(1)}$ once the eikonal approximation is applied to the internal light-quark propagator (m is the light-quark mass):

$$\frac{i(\not q_2 - \not k - \not l)}{(q_2-k-l)^2 - m^2} \to \frac{i\not q_2}{-2q_-(k_+ + l_+)}, \tag{13}$$

where l denotes the gluon momentum. In the collinear region of phase space, the eikonal approximation (13) is valid: thus, the IR singularities in $F_\mu^{(1)}$ and $\Phi^{(1)}\otimes T^{(0)}$ from Fig. 2a are equal and cancel in (12).

The weak-vertex correction, Fig. 2b, yields no IR singularities either for $F_\mu^{(1)}$ or $\Phi^{(1)}\otimes T^{(0)}$, but it contains large single and double (Sudakov) logarithms of $\Lambda_{\mathrm{QCD}}/m_b$. The box diagram, Fig. 2c, contributes at leading-twist to $F_\mu^{(1)}$ only in the region where the gluon has a soft momentum $O(\Lambda_{\mathrm{QCD}})$. But in this region, the eikonal approximation can be applied and leads to the same integral as $\Phi^{(1)}\otimes T^{(0)}$. Therefore, the two contributions are identical, and the box diagram does not contribute to $T^{(1)}$.

In all diagrams, IR singularities cancel between $F_\mu^{(1)}$ and $\Phi^{(1)} \otimes T^{(0)}$ [3] – factorisation holds at one loop:

$$F_{A,V}(E_\gamma) = \int d\tilde{k}_+ \Phi_+^B(\tilde{k}_+;\mu_F) T(\tilde{k}_+, E_\gamma; \mu_F)\,, \qquad (14)$$

$$T = -\frac{f_B M_B Q_u}{3\sqrt{2}E_\gamma}\frac{1}{\tilde{k}_+}\left[1 + \frac{\alpha_s C_F}{4\pi} K(\tilde{k}_+, E_\gamma; \mu_F)\right] \qquad (15)$$

$$\begin{aligned} K = &\log^2 \frac{2\tilde{k}_+ q_-}{\mu_F^2} - \frac{1}{2}\log^2\frac{m_b^2}{\mu_F^2} + \frac{5}{2}\log\frac{m_b^2}{\mu_F^2} - 2\log\frac{m_b}{\sqrt{2}q_-} \\ &+2\log\frac{m_b^2}{\mu_F^2}\log\frac{m_b}{\sqrt{2}q_-} - \frac{\sqrt{2}q_-}{m_b - \sqrt{2}q_-}\log\frac{\sqrt{2}q_-}{m_b} \qquad (16) \\ &+2\mathrm{Li}_2\left(1 - \frac{\sqrt{2}q_-}{m_b}\right) + 4\mathrm{Li}_2\left(1 - \frac{m_b}{\sqrt{2}q_-}\right) - \frac{\pi^2}{4} - 7\,. \end{aligned}$$

1.3 Higher orders

Equation (16) involves large double and single logarithms for $\mu_F^2 = O(\Lambda_{\mathrm{QCD}} m_b)$. They can be resummed in the framework of the Soft-Collinear Effective Theory (SCET), which describes the interaction of infinitely heavy quarks with soft and/or collinear quarks and gluons [6]. The weak $b \to u$ current can be matched onto the operators O_i of the effective theory [7]: $\bar{u}\gamma^\mu(1-\gamma_5)b = \sum_i C_i^{\mathrm{SC}}(\mu) O_i^\mu(\mu)$.

The C_i are Wilson coefficients which describe the physics above μ. The large logarithms contained in the C_i can be resummed using renormalisation group equations:

$$C_i^{\mathrm{SC}}(\mu_F) = C_i^{\mathrm{SC}}(m_b)\exp[-S(E_\gamma;\mu_F)]\,. \qquad (17)$$

$C_i^{\mathrm{SC}}(m_b)$ is a perturbative series in powers of $\alpha_s(m_b)$. S contains large (Sudakov) logarithms of μ_F/m_b and does not depend on the SCET operator considered. For this decay, only the SCET operator O_3 in [7] is relevant.

In the SCET expression for the weak-current contribution to F_μ, one can identify the soft-gluon interaction below μ_F to the contribution to $\Phi \otimes T$, which has to be removed to determine the hard-scattering kernel. The latter corresponds therefore to the sum of the remaining collinear-gluon interaction and the electromagnetic-current [3]:

$$T = -C_3^{\mathrm{SC}}(\mu_F)\frac{f_B Q_u m_b}{3\sqrt{2}E_\gamma}\frac{1}{\tilde{k}_+}\left[1 + \frac{\alpha_s(\mu_F)\,C_F}{4\pi}K_t\right] \qquad (18)$$

$$K_t = \log^2\frac{2q_-\tilde{k}_+}{\mu_F^2} - \frac{\pi^2}{6} - 1\,. \qquad (19)$$

The large logarithms in (14) are now exponentiated in the matching coefficient $C_3^{\mathrm{SC}}(\mu_F)$ for $\mu_F^2 = O(\Lambda_{\mathrm{QCD}} m_b)$.

2 $B \to \gamma\gamma$ and $B \to \gamma\ell^+\ell^-$

The discussion is very similar for $B \to \gamma\gamma$ and $B \to \gamma\ell^+\ell^-$ (with $(1-2E_\gamma/m_b) \gg \Lambda_{\mathrm{QCD}}/m_b$, i.e., away from the endpoint). One can show that these decays can be expressed, at leading twist, in terms of the axial/vector form factors $F_{V,A}$ defined above and the tensor form factors $F_{T,T'}$ [4], stemming from the operators O_7, O_9, O_{10} of the effective Hamiltonian. Other operators contribute to these decays, but power counting arguments show that they affect only short distances at leading twist (their contributions can thus be included in effective Wilson coefficients for O_7, O_9, O_{10}).

$F_{T,T'}$ can be discussed along similar lines as $F_{V,A}$. The only difference comes from the weak-vertex correction, and arises only from the (ultraviolet) dynamics above m_b. This is checked at one loop, and can be generalised at higher orders within SCET [4]: $F_{T,T'}$ can be expressed exactly as in eqs. (18)-(19) up to replacing $C_3^{\mathrm{SC}} \to C_9^{\mathrm{SC}}$. C_9^{SC} fulfills (17) with the *same* exponential term S.

Thus, the nonperturbative effects encoded in the moments of the B-meson LCDA are identical for all 3 decays. Ratios of $F_{T,T'}$ and $F_{V,A}$ and thus those of decay amplitudes for radiative B-decays are proportional to $C_9^{\mathrm{SC}}(m_b)/C_3^{\mathrm{SC}}(m_b)$ which can be expressed as a perturbative series in $\alpha_s(m_b)$.

3 Conclusion

We have shown that QCD factorisation holds at one loop for radiative B-decays. Using SCET, large (Sudakov) logarithms were resummed, and the three decays were shown to differ from each other only through (perturbative) dynamics from scales above m_b. Further extensions of the proof were presented in [8]. The present results for purely radiative B-decays could be useful to understand more complicated processes with hadronic final states.

Acknowledgements. I would like to thank Prof. C.T. Sachrajda for our close collaboration on the topics reported here. LPT is an Unité mixte de recherche du CNRS et de l'Université Paris XI (UMR 8627).

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